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Implication of Brane fluctuations to indirect collider signals

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Abstract: We study the effect of brane fluctuation on the indirect signals of high energy colliders. Brane fluctuation could act as a regulator of divergent expression of infinite tower of Kaluza-Klein graviton effects. The phenomenological parameter λ, introduced by Hewett, is shown to be determined in our setting, and its dramatic behaviors depending on the $D = 4 + \delta$ dimensional gravitation scale M_D, 'softening parameter' Δ, and $\sqrt{s}$ of collider are presented. The present exclusion bounds from the processes $e^+e^- \to \gamma\gamma$ and $p\bar{p} \to e^+e^-, \gamma\gamma(\gamma)$ are considered within the parameter space (M_D, Δ) with respect to the number of extra dimensions.

Keywords: Beyond Standard Model, Compactification and String Models.

Contents

1. Introduction

It is phenomenologically interesting when the size of extra dimension is so large that the Kaluza-Klein excitations of bulk fields could directly affect the low energy phenomena . If all the standard model(SM) fields are confined on the brane and only graviton can propagate through the bulk, the size of extra dimension can be large enough to nullifying the hierarchy between the weak scale and the Planck scale [1, 2]. In that case, the Kaluza-Klein (KK) tower of graviton can make sizable contributions to the collider physics [3, 4]. Not only signals with direct production of KK graviton, but indirect signals with the KK mediation can provide chances to detect the effects from the extra dimensions. The scattering cross section including KK graviton mediating diagrams can be written as

$$\sigma_{\text{Total}} = \sigma_{\text{SM}} + \eta_{KK}\sigma_{\text{Mix}} + \eta_{KK}^2\sigma_{KK} \,. \tag{1.1}$$

Where η_{KK} denotes the propagating factor of KK graviton tower defined as

$$\eta_{KK} \equiv \frac{i^2}{8M_P^2}\sum_{KK}\frac{1}{s-m_{KK}^2} \tag{1.2}$$

when only s-channel diagrams are involved. Here $m_{KK}^2 \equiv \vec{n}\cdot\vec{n}/R^2$ is the mass of the KK graviton in state $\vec{n} = (n_1, n_2, \ldots, n_\delta)$, the factor $1/8$ is introduced for future convenience and the Planck scale M_P is for gravitational coupling. The case with t-, u- channel KK mediating diagrams also contributing like $e^+e^- \to e^+e^-$ can be considered as a straight-forward ways. The summation through the whole tower of KK states is generally divergent and so we should take a cutoff scale ($N_\Lambda = \Lambda/R$) to get finite result and then we simply throw the rests away [5] (see, for loop calculations of Kaluza-Klein states [6, 7]). The size of extra dimension R is related to the scales of the gravitation as $R^\delta M_D^{2+\delta} = M_P^2$, where

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δ is number of extra dimensions. With small spacing ($\sim 1/R$) the summation could be approximated to the integration, we can obtain η_{KK} at the limit $\Lambda/\sqrt{s} \gg 1$ as

$$\begin{aligned} \eta_{KK}(\text{Rigid}) &\approx \frac{\pi}{2M_D^4} \ln\left(\frac{\Lambda}{\sqrt{s}}\right) \qquad (\delta = 2)\,, \\ &\approx \frac{\Omega_\delta s^{\delta/2-1}}{8M_D^{\delta+2}} \left(\frac{\Lambda}{\sqrt{s}}\right)^{\delta-2} \qquad (\delta > 2)\,, \end{aligned} \tag{1.3}$$

where Ω_δ is the solid angle in δ dimensional space, e.g., $\Omega_2 = 2\pi$ in 2 dimensional space.

By taking $\Lambda \sim M_D$, the factor could be estimated as $\eta_{KK} \sim \lambda/M_D^4$, where λ encapsulates all the uncertainties from the number of extra dimensions, the unknown relation between Λ and M_D, and threshold effects coming from the string theory beyond the cutoff scale [8]. Though it is obvious that λ is dependent on the number of extra dimensions and the energy scale of collider $\sqrt{s}$, it is usually assumed that the value is $\mathcal{O}(1)$ and insensitive to the number of extra dimensions. With this assumptions, indirect signals of extra dimensions have been treated independently of δ etc. [9, 10]. However from a more close study including the dynamics of the brane fluctuation, λ shows dramatic behavior depending δ, $\sqrt{s}$ and softening scale parameter which is essentially determined by tension of the brane [11, 12]. This paper is organized as following. In the section 2, we first present the explicit formula for η_{KK} in terms of the brane tension, number of extra dimensions and the C.M. energy of the colliding particles. Using the formula we can find the appropriate expression for the phenomenological parameter λ in terms of the tension parameter. From the expressions we can 'translate' the experimental bounds from LEP-II and Tevatron in terms of brane tension scale and M_D scale in section 3. Summary and conclusion of our study will be given in section 4.

2. Brane Fluctuations and determining λ parameter

In the string theory embedding of large extra dimensional theory, our world might be a dynamical object carrying finite tension [13]. It is very natural since any relativistic consideration does not allow strictly rigid objects. Thus the formalism including brane fluctuation must ultimately be employed to probe the high energy physics of extra dimensions with brane [5, 14]. The brane fluctuation could be described by introducing Nambu-Goldstone boson $\vec{\phi}(x)$ which came from the spontaneous translational symmetry breaking. The dynamics of the Nambu-Goldstone boson, inducing the metric on the fluctuating brane, is described by the Nambu-Goto action with the induced metric.

$$\mathcal{L} = -\tau \int d^4x \sqrt{-g} = -\tau \int d^4x \left(1 - \frac{1}{2}\partial_\mu \vec{\phi}(x) \cdot \partial^\mu \vec{\phi}(x) + \cdots \right), \tag{2.1}$$

where the tension of the brane is denoted as τ. After expanding the bulk graviton field around the compact extra dimensions and taking normal ordering for the expansion modes in perturbation framework, the interaction lagrangian with the KK gravitons and the SM particles is shown to carry an exponential 'softening factor' [15, 16]:

$$\mathcal{L} \supset -\frac{1}{M_P} g_{\mu\nu} T^{\mu\nu}(\text{SM}) \Rightarrow -\frac{1}{M_P} \sum_{\vec{n}} e^{-\frac{1}{2}m_{KK}^2/\Delta^2} g_{\mu\nu}^{\vec{n}} T^{\mu\nu}(\text{SM})\,, \tag{2.2}$$

where $\frac{1}{\Delta^2}$ is the free propagator of $\vec{\phi}$,

$$\frac{1}{\Delta^2} \equiv \langle \phi(x)\phi(y)\rangle|_{x\to y} = -\frac{1}{4\pi^2\tau}(x-y)^{-2}|_{x\to y} \text{ [14]}. \tag{2.3}$$

In principle, the scale Δ^2 could be determined by the tension of the brane and the cutoff scale of loops of a $\phi(x)$ field. In this study we just take the scale Δ as a free parameter of the theory which shows the effect of the brane fluctuation. Note that when the scale is chosen at infinity, the action reduced to the normal UN-fluctuating case and if it is chosen at the same order of M_D scale it will provide very rich phenomenology of high energy colliders. Since the coupling of the higher KK states are quite suppressed with the exponential softening factor, divergent expression for the infinite KK graviton contribution could be naturally regularized.

For the indirect collider signals, we can get the regularized expression for η_{KK} or λ parameter as

$$\eta_{KK}(\text{Fluctuating}) \Rightarrow \frac{-1}{8M_P^2}\int d^\delta n \frac{e^{-\vec{n}\cdot\vec{n}/(R^2\Delta^2)}}{s-\vec{n}\cdot\vec{n}/R^2}. \tag{2.4}$$

From the above relation we can derive the expression for the parameter λ as follows.

$$\lambda(\text{Fluctuating}) = \frac{-\Omega_\delta}{8}(\frac{\sqrt{s}}{M_D})^{\delta-2}\mathcal{I}\left(\delta, \frac{s}{\Delta^2}\right). \tag{2.5}$$

The integral function is introduced as

$$\mathcal{I}(\delta, s/\Delta^2) = \int dx \frac{x^{\delta-1}e^{-x^2(s/\Delta^2)}}{1-x^2} \tag{2.6}$$

and we take the principal value for the singular integral without pole contribution of KK state. The value for the integration is very stable with respect to the UV cutoff and we understand the behavior with exponential suppression factor for the large x region. This is very general behavior from the brane fluctuation working as physical regularization factor. In figure 1–3, we present the numerical results for the parametric dependence of λ parameter. Note that the sign of λ is essentially determined by the ratio of (s/Δ^2) for given δ. If the ratio is large enough that the contribution of the light KK gravitons are quite suppressed, the integral function surely gives negative sign. In that case, λ could have positive sign. But for the case with the large softening parameter($\Delta > \mathcal{O}(1)$ TeV), λ usually carries negative sign and we would like to concentrate on this possibility in great detail.

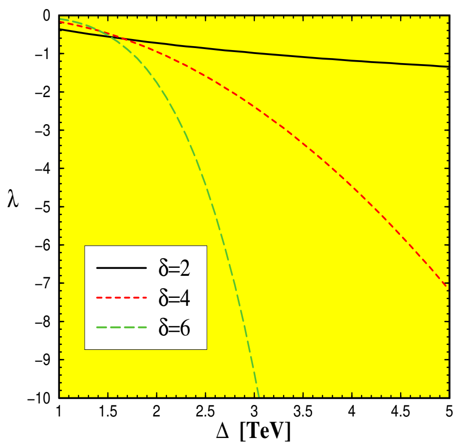

Figure 1: Softening parameter Δ dependence of λ. The solid line, dotted line and dashed line denote the case for $\delta = 2, 4$ and 6, respectively.(All the following graphes take the same convention for δ lines. M_D and $\sqrt{s}$ are set as 2 and 0.5 TeV, respectively.

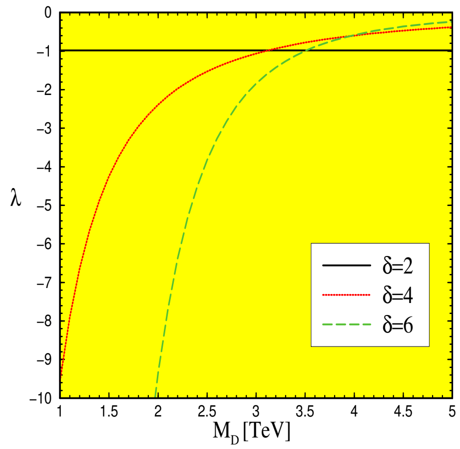

Figure 2: M_D dependence of λ with varying $\delta = 2, 4$ and 6. Δ and $\sqrt{s}$ are set as 3 and 0.5 TeV, respectively.

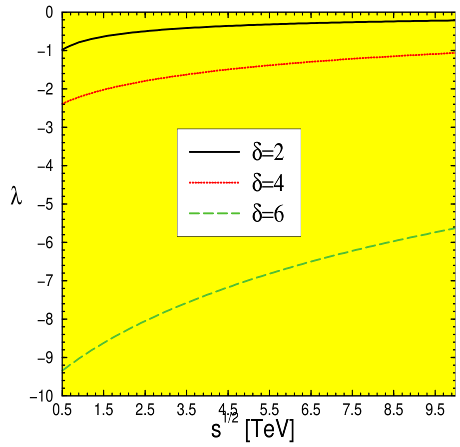

Figure 3: The collider C.M. energy $\sqrt{s}$ dependence of λ with respect to the varying $\delta = 2, 4$ and 6. M_D and Δ are set as 2 and 3 TeV, respectively.

Figure 1 shows the softening parameter dependence of the λ parameter when M_D and $\sqrt{s}$ are set to be 2 and 0.5 TeV, respectively. The δ dependence is very crucial. It should be noted that within the reasonable range of the parameter space, λ value is well approximated as $\mathcal{O}(1)$ value for the case with $\delta = 2$ and 4. But for the case with $\delta = 6$, the absolute value can be much larger.

In figure 2, we plot the M_D dependence when Δ and $\sqrt{s}$ are set to be 3 and 0.5 TeV, respectively. The absolute value is suppressed with larger M_D. We can understand this behavior from the inversely depending relation given above. The case for $\delta = 2$ is very stable with respect to varying M_D but it is not general behavior for larger dimensions. We can also see that the absolute value is $\mathcal{O}(1)$ for the case with two or four extra dimensions but can be very large when $\delta = 6$. Crossing occurs at $M_D \approx 3$ TeV and which is generally possible from the non-linear dependence of the parameters.

In figure 3, $\sqrt{s}$ dependence is plotted when M_D and Δ are set to be 2 and 3 TeV, respectively. We can see the rather smooth dependence for two or four extra dimensional cases but the slope is a bit steeper with six extra dimensions. With given parameter set, the λ has $\mathcal{O}(1)$ value in the parameter space.

3. Collider bounds

Now let us consider the experimental bounds for the indirect signals of KK gravitons from the fluctuating brane. Here we consider the dielectron and diphoton production processes as a concrete example. However, our method is quite general so that we can extend the study to any indirect collider signals of extra dimensions with brane fluctuation regularization. Usually experimental bounds for the extra dimensions are given by M_D assuming $\lambda = \pm 1$. However,as was seen at the above section, the parameter λ generally depends on not only M_D but also δ and Δ with brane fluctuation effects. We could only impose experimental bounds within the parameter space of three dimensions:(M_D, Δ, δ) with given center of mass energy of collider.

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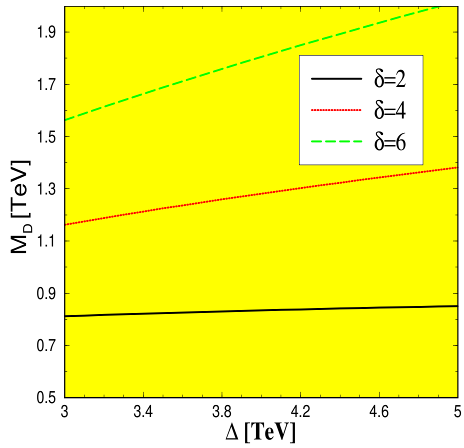

Figure 4: 95 % C.L. M_D exclusion bound from the LEPII data with the process $e^+e^- \to \gamma\gamma(\gamma)$ with C.M. energies from 189 to 202 GeV.

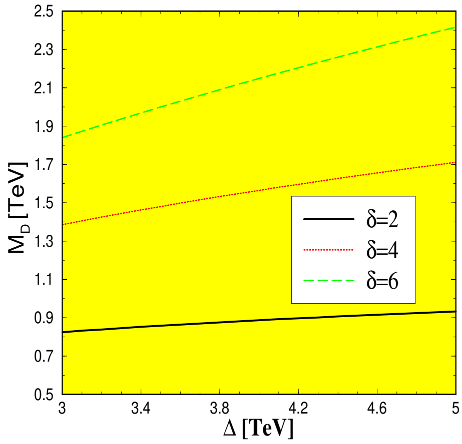

Figure 5: 95 % C.L. exclusion bounds from the Tevatron data with the processes $p\bar{p} \to e^+e^-, \gamma\gamma$ detected by D0 detector. C.M. energy is 1.8 TeV.

We first consider the LEP-II bound from the $e^+e^- \to \gamma\gamma(\gamma)$ [17, 18] at center of mass energy ranging from 189 GeV to 202 GeV by DELPHI. The bound was given as $M_D > 713$ GeV with $\lambda = 1$ and $M_D > 691$ GeV with $\lambda = -1$ (95% C.L.)[19]. As was commented, we only consider the case with negative λ case. In figure 4, the exclusion bound is presented. For the case with $\delta = 2$, our new bound is $M_D > 810$ GeV and the bound is almost independent of Δ scale. Generally the exclusion bound is larger with larger δ and increasing with larger Δ.

Now let us consider the Tevatron bound from the diphoton and dielectron pair production processes [20] obtained by D0. The center of mass energy is taken about 1.8 TeV. The lower limits at 95 % C.L. on the M_D scale was given between 1.0 and 1.1 TeV with $\lambda = 1$ and -1, respectively [21]. In figure 5, we see the exclusion bound for M_D scale with respect to the softening scale and number of extra dimensions for the data. The bound for $\delta = 2$ case is similar with LEP-II bound but in the cases with larger number of extra dimensions the bounds are much higher. This behavior could be understood by C.M. energy dependence of λ parameter in eq. (2.5) and (2.6).

4. Summary and conclusion

We study the implication of the brane fluctuation to the indirect signals of collider physics. From the exponential softening factor the brane fluctuation provide very natural regularization scheme for KK states summation. With this regularization we can determine λ parameter with respect to parameters of M_D, Δ, $\sqrt{s}$ and δ. The experimental exclusion bounds of existing data and sensitivity bounds for future colliders could be determined within the parameter space of (M_D, Δ, δ). As a concrete example, we found the exclusion bounds from the LEP-II diphoton production data with C.M. energy from 189 to 202 GeV and Tevatron dielectron and diphoton production data with C.M. energy of 1.8 TeV.

The brane fluctuation provides a new setup for phenomenology of extra dimension searches. The indirect signals of extra dimensions, the data is shown to be highly dependent on not only M_D scale but also δ and Δ. All the sensitivity bounds should be determined within the parameter set of (M_D, Δ, δ) if we consider the brane fluctuation.

Finally, let us briefly comment on the brane fluctuation regularization within the Randall-Sundrum setting [22]. Unfortunately, it is not possible to introduce the brane fluctuation regularization in Randall-Sundrum's case. In the RS case, our TeV brane carries negative tension to make the warped geometry and the branes reside on the orbifold fixed points. Thus we cannot apply any regularization of brane fluctuation. Just sharp cutoff at reasonable range could provide a finite results of low energy observables by assuming only KK modes up to the cutoff scale is responsible for calculation (see e.g., [23, 24]). Further study is needed to formulate natural regularization formalism with the negative tension brane on the orbifold fixed point.

Acknowledgments

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